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Scalar-Gauss–Bonnet gravity and cosmic acceleration: Comparison with quintessence dark energy

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The late-time cosmic acceleration of the universe is explained in the framework of the Einstein-scalar-Gauss–Bonnet (GB) theory by considering different appropriate forms for the GB coupling parameter and the scalar field. The physical quantities such as the potential energy, the GB coupling parameter, the energy density, the pressure and the deceleration parameter are obtained in an exact parametric form of the volume scale factor of the universe. The behavior of the deceleration parameter shows a transition from the deceleration phase to the acceleration phase at the late times in agreement with the observational data. Finally, the GB energy density ratio is compared to the matter energy density and the scalar (the quintessence) field energy density in the early and late times.

Keywords: Gauss–Bonnet; cosmic acceleration; dark energy; quintessence.

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1. Introduction

Despite two decades after announcing current accelerated state of our expanding universe based on observational data reported in Refs. 1 and 2 and a number of research efforts to identify the origin of this late-time cosmic acceleration of the universe now known as dark energy, there isn't a standard explanation for such a different energy from the ordinary matter species such as baryons and radiation, in a sense that it has a negative pressure (see e.g. the reviews Refs. 3–9). Among a large number of possible mechanisms such as the cosmological constant, scalar fields, extra dimensions theories, modifications of general relativity and other alternatives, the simplest candidate for dark energy is the so-called cosmological constant Λ, whose energy density remains constant.[7] Although a tiny positive cosmological constant may explain the current acceleration of the universe, it would encounter many theoretical difficulties including the fine-tuning and the coincidence problems.[10]

There are two approaches to construct models of dark energy without considering the cosmological constant.[11] The first approach is to modify the right-hand side of the Einstein equations by considering specific forms of the energy–momentum tensor $T_{\mu\nu}$ with a negative pressure. The representative models corresponding to this class are the so-called cosmon or quintessence,[12–19] k-essence,[20,21] and perfect fluid models.[22,23] The second approach for the construction of dark energy models is to modify the left-hand side of the Einstein equations. The representative models corresponding to this class are the so-called $f(R)$ gravity (modified gravity) theories,[24–26] scalar–tensor theories,[27–31] and braneworld models.[32–35] A class of modified gravity theories is to add a general function of the Ricci scalar, Ricci tensor and the Riemann tensors, e.g. $f(R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \ldots)$ to the Einstein–Hilbert action. However, there is a quadratic combination of the Riemann curvature tensor called Gauss–Bonnet (GB) term that keeps the equations at second-order in the metric.[36,37] Of course, without GB modification, one can obtain a second-order equation in metric in Palatini framework. One example of this is in loop quantum cosmology.[38] Recently, an inverse procedure has been developed to derive Hamiltonians and actions (without any prior knowledge) which lead to modified gravity theories such as loop quantum cosmology, braneworlds etc., for different signs of modified terms in Friedmann and Raychaudhuri equations.[39] The GB term is a topological invariant quantity in four-dimensions and dynamically irrelevant; however, in two cases, it is no longer a total derivative and therefore there is no reason to leave it out. The first one corresponds to the higher-dimensional theories such as braneworld models, the second one corresponds to the scalar–tensor theories where the GB term couples to a scalar field and the four-dimensional (4D) gravity is modified. It is noticeable that the GB term naturally arises as a correction to the tree-level action of low-energy effective string theory.[40,41] The role of the GB term coupled to the scalar field on the cosmic acceleration of the universe has been already extensively investigated.[42–74] The possibility of crossing the phantom divide line through GB interaction has been explored too.[44,47,56,68,75–78]

In the present paper, using the same method as in Ref. 13, we want to study the possibility of realizing the late-time cosmic acceleration of the universe in the presence of the GB term coupled to a scalar field. In the next section, the field equations corresponding to the model under consideration are derived and then the general solutions in the scalar-GB gravity are obtained. Then, by a suitable choice of the scalar field as an arbitrary function of volume scale factor, the exact solutions of the gravitational field equations are found. Finally, both analytically and graphically, it is briefly discussed about obtaining results with the aim of possible explanation of the current accelerating expansion of the universe and with a comparison to the quintessence dark energy in the early and late times.

2. General Solutions of the Field Equations in Scalar-GB Gravity

Consider a 4D action including the Einstein–Hilbert term and a scalar field ϕ coupled nonminimally to gravity via a general coupling parameter $\xi(\phi)$ to the quadratic GB term. Such a theory is described by the following action[45]

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa_4^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi) - \xi(\phi)R^2_{\rm GB} + \mathcal{L}_m\right], \tag{1}$$

where the potential energy of the scalar field U and the coupling parameter of the GB term ξ are functions of the scalar field ϕ and $\mathcal{L}_m$ is the Lagrangian density of the matter field. The GB term $R^2_{\rm GB}$ is defined as

$$R^2_{\rm GB} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \tag{2}$$

By varying the action (1) with respect to the metric $g_{\mu\nu}$, the corresponding gravitational field equation is found as

$$\begin{aligned}\frac{1}{\kappa_4^2}\left(R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu}\right) = {} & T^{\mu\nu(m)} + \frac{1}{2}T^{\mu\nu(\phi)} + \frac{1}{2}g^{\mu\nu}\xi(\phi)R^2_{\rm GB} - 2\xi(\phi)RR^{\mu\nu} \\ & + 4\xi(\phi)R^\mu_\rho R^{\nu\rho} - 2\xi(\phi)R^{\mu\rho\sigma\tau}R^\nu_{\rho\sigma\tau} \\ & + 4\xi(\phi)R^{\mu\rho\nu\sigma}R_{\rho\sigma} + 2(\nabla^\mu\nabla^\nu\xi(\phi))R \\ & - 2g^{\mu\nu}(\nabla^2\xi(\phi))R - 4(\nabla_\rho\nabla^\mu\xi(\phi))R^{\nu\rho} - 4(\nabla_\rho\nabla^\nu)R^{\mu\rho} \\ & + 4(\nabla^2\xi(\phi))R^{\mu\nu} + 4g^{\mu\nu}(\nabla_\rho\nabla_\sigma\xi(\phi))R^{\rho\sigma} \\ & - 4(\nabla_\rho\nabla_\sigma\xi(\phi))R^{\mu\rho\nu\sigma},\end{aligned} \tag{3}$$

where $T^{(m)}_{\mu\nu}$ represents the energy–momentum tensor of the matter field. We assume that the matter context of the universe is a fluid represented by the following energy–momentum tensor

$$T^{(m)}_{\mu\nu} = (\rho_m + p_m)u_\mu u_\nu + p_m g_{\mu\nu}, \qquad u_\mu u^\mu = -1, \tag{4}$$

here ρ_m and p_m are the energy density and pressure of the fluid given by $p_m = (\gamma - 1)\rho_m$, $(0 < \gamma < 2)$. For the scalar field ϕ, with the Lagrangian density $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi)$, the energy–momentum tensor is given by[55]

$$T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + U(\phi)\right). \tag{5}$$

Assuming that the line-element is described by the spatially-flat Friedmann–Robertson–Walker (FRW) metric

$$ds^2 = -dt^2 + a(t)^2\sum_{i=1}^{3}(dx^i)^2, \tag{6}$$

the equations of motion become

$$3H^2 = \rho_\phi + \rho_{\rm GB} + \rho_m, \tag{7}$$

$$2\dot{H} + 3H^2 = -p_\phi - p_{\rm GB} - p_m, \tag{8}$$

where the energy density and pressure of the GB term and the quintessence field are defined as

$$\rho_{\rm GB} = 24H^3\dot{\xi}(\phi), \tag{9}$$

$$p_{\rm GB} = -8H^2\ddot{\xi}(\phi) - 16H(\dot{H} + H^2)\dot{\xi}(\phi), \tag{10}$$

and

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + U(\phi), \tag{11}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - U(\phi). \tag{12}$$

The scalar field equation is

$$\ddot{\phi} + 3H\dot{\phi} + U'(\phi) + 24\xi'(\phi)(\dot{H}H^2 + H^4) = 0, \tag{13}$$

where the prime denotes differentiation with respect to ϕ. Now, from Eqs. (7) and (8), we obtain

$$2\dot{H} + \dot{\phi}^2 + \gamma\rho_m + 8H^3\dot{\xi}(\phi) - 8H^2\ddot{\xi}(\phi) - 16H\dot{H}\dot{\xi}(\phi) = 0. \tag{14}$$

Considering the volume scale factor of the universe and the Hubble parameter in terms of it

$$V(t) = a(t)^3, \quad H = \frac{\dot{a}}{a} = \frac{\dot{V}}{3V}, \tag{15}$$

the gravitational field equation (15) takes the following form (14)

$$\frac{\ddot{V}}{V} + \frac{3}{2}\gamma\rho_0 V^{-\gamma} + \frac{3}{2}\dot{\phi}^2 - \left[1 + \frac{4}{3}\ddot{\xi}(\phi)\right]\left(\frac{\dot{V}}{V}\right)^2 - \frac{8}{3}\left(\frac{\dot{V}\ddot{V}}{V^2}\right)\dot{\xi}(\phi) + \frac{28}{9}\left(\frac{\dot{V}}{V}\right)^3\dot{\xi}(\phi) = 0. \tag{16}$$

For $\xi(\phi) = 0$, the above equation reduces to the quintessence model.[13] Introducing $u = \dot{V}$, the differential equation describing the dynamics of the universe is found as

$$\frac{du^2}{dV} - \frac{4}{V}\frac{d\xi(\phi)}{dV}u^2\frac{du^2}{dV} + \left[3V\left(\frac{d\phi}{dV}\right)^2 - \frac{2}{V}\right]u^2 + \left[\frac{56}{9}\frac{1}{V^2}\frac{d\xi(\phi)}{dV} - \frac{8}{3V}\frac{d^2\xi(\phi)}{dV^2}\right]u^4 = -3\gamma\rho_0 V^{1-\gamma}. \tag{17}$$

To solve this equation, we specify ξ as a function of V. By choosing a model in which $\frac{d\xi}{dV} = \alpha\frac{V}{u^2}$, where α is a positive constant, Eq. (17) can be transformed to the form

$$\left(\frac{3-4\alpha}{3}\right)\frac{du^2}{dV} + \left[3V\left(\frac{d\phi}{dV}\right)^2 - \left(\frac{18-32\alpha}{9}\right)\frac{1}{V}\right]u^2 = -3\gamma\rho_0 V^{1-\gamma}, \tag{18}$$

which leads to the following solution

$$u^2 = V^{\frac{32\alpha-18}{3(4\alpha-3)}} e^{\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} \times \left[\int \frac{9\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right], \tag{19}$$

where c is a constant of integration. Substituting $u = \dot{V}$, it is found that

$$t - t_0 = \int \frac{V^{-\frac{32\alpha-18}{6(4\alpha-3)}} e^{-\frac{9}{2(4\alpha-3)}\int V\left(\frac{d\phi}{dV}\right)^2 dV}}{\left[\int \frac{9\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right]^{1/2}} dV, \tag{20}$$

where t_0 is a constant of integration. Therefore, the general solutions of the gravitational field equations can be written as in the following exact parametric forms

$$U(V) = V^{\frac{32\alpha-18}{3(4\alpha-3)}} e^{\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} \times \left[\int \frac{9\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right] \times \left[-\frac{1}{2}\left(\frac{d\varphi}{dV}\right)^2 + \frac{V^{-2}}{3}\left(1 - \frac{8\alpha}{3}\right)\right] - \rho_0 V^{-\gamma}, \tag{21}$$

$$\xi(V) = \int \frac{\alpha V dV}{V^{\frac{32\alpha-18}{3(4\alpha-3)}} e^{\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} \times \left[\int \frac{9\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right]}, \tag{22}$$

$$\rho_\phi(V) = V^{\frac{32\alpha-18}{3(4\alpha-3)}-2} e^{\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} \times \left[\int \frac{3\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right] - \rho_0 V^{-\gamma} - \rho_{\rm GB}(V), \tag{23}$$

$$p_\phi(V) = V^{\frac{32\alpha-18}{3(4\alpha-3)}} e^{\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} \times \left[\int \frac{9\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right] \times \left[-\frac{3}{(4\alpha-3)}\left(\frac{d\phi}{dV}\right)^2 + \frac{(9-20\alpha)}{9(4\alpha-3)V^2}\right] + \left[1 - \frac{4\alpha\gamma}{(4\alpha-3)}\right]\rho_0 V^{-\gamma} - p_{\rm GB}(V), \tag{24}$$

$$\rho_{\rm GB}(V) = \alpha V^{\frac{32\alpha-18}{3(4\alpha-3)}-2} e^{\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} \times \left[\int \frac{8\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right], \tag{25}$$

$$p_{\rm GB}(V) = \alpha V^{\frac{32\alpha-18}{3(4\alpha-3)}} e^{\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} \times \left[\int \frac{9\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right] \times \left[\frac{4}{3(4\alpha-3)}\left(\frac{d\phi}{dV}\right)^2 + \frac{4(12-8\alpha)}{81(4\alpha-3)}\frac{1}{V^2}\right] - \frac{4\alpha}{3(3-4\alpha)}\gamma\rho_0 V^{-\gamma}, \tag{26}$$

$$q(V) = \left[3V^2\left(\frac{d\phi}{dV}\right)^2 + \frac{8\alpha}{9} + \frac{3\gamma\rho_0 V^{(2-\gamma)} e^{\frac{-9}{(4\alpha-3)}\int V\left(\frac{d\phi}{dV}\right)^2 dV}}{V^{\frac{32\alpha-18}{3(4\alpha-3)}}\left(\int \frac{9\gamma\rho_0}{(4\alpha-3)} V^{1-\gamma-\frac{32\alpha-18}{3(4\alpha-3)}} e^{-\frac{9}{(4\alpha-3)}\int V(\frac{d\phi}{dV})^2 dV} dV + c\right)}\right] \times \frac{9}{2(3-4\alpha)} - 1, \tag{27}$$

where $q(V)$ is the deceleration parameter of the universe in terms of the volume scale factor and in the limit of vanishing the GB term, ($\alpha = 0$), the above equations reduce to the Eqs. (9)–(12) in Ref. 13.

In the next section, by suitable choice of the scalar field ϕ as an arbitrary function of volume scale factor, some exact classes of solutions of the gravitational field equations are studied.

3. Accelerated Expansion of the Universe from Scalar-GB Gravity

Now let us study the role of the GB term coupled to the scalar field in the current acceleration of the present universe, i.e. $p_m = 0$. For this end, we should specify the scalar field ϕ as an arbitrary function of the volume factor V. Choosing $\phi = \sqrt{\beta}\ln V$ where β is a positive constant,[79–81] leads to the following forms of Eqs. (21)–(27)

$$U(V) = \left[-\frac{\beta}{2} + \frac{1}{3}\left(1 - \frac{8\alpha}{3}\right)\right] f(V) - \rho_0 V^{-1}, \tag{28}$$

$$\xi(V) = \int \frac{\alpha(3-4\alpha)dV}{3Vf(V)}, \tag{29}$$

$$\rho_\phi(V) = \frac{1}{(3-4\alpha)} f(V) - \rho_0 V^{-1} - \rho_{\rm GB}(V), \tag{30}$$

$$p_\phi(V) = \left[\frac{(20\alpha + 27\beta - 9)}{3(4\alpha-3)^2}\right] f(V) + \left[1 - \frac{4\alpha}{(4\alpha-3)}\right] \rho_0 V^{-1} - p_{\rm GB}(V), \tag{31}$$

where

$$\rho_{\rm GB}(V) = \frac{8\alpha}{3(3-4\alpha)} f(V), \tag{32}$$

$$p_{\rm GB}(V) = -\frac{4\alpha(12 - 18\alpha + 27\beta)}{27(4\alpha-3)^2} f(V) + \frac{4\alpha}{3(4\alpha-3)} \rho_0 V^{-1}, \tag{33}$$

and

$$q(V) = \frac{9}{2}\left[-\frac{(8\alpha + 27\beta)}{9(4\alpha-3)} + \frac{\rho_0}{V f(V)}\right] - 1, \tag{34}$$

where we have defined

$$f(V) = c\left(\frac{3-4\alpha}{3}\right) V^{\frac{32\alpha+27\beta-18}{3(4\alpha-3)} - 2} + \frac{3\rho_0 V^{-1}}{\left[-1 + \dfrac{32\alpha + 27\beta - 18}{3(4\alpha-3)}\right]}. \tag{35}$$

To avoid the singular behavior of the function $f(V)$, we should take $\alpha \neq \frac{3}{4}$. For $\xi(\phi) = 0$, we have $\alpha = 0$ and thus $\rho_{\rm GB} = p_{\rm GB} = 0$ and Eqs. (28)–(35) reduce to the following Eqs. in Ref. 13

$$U(V) = \left(\frac{1}{3} - \frac{\beta}{2}\right) f(V) - \rho_0 V^{-1}, \tag{36}$$

$$\rho_\phi(V) = \frac{1}{3} f(V) - \rho_0 V^{-1}, \tag{37}$$

$$p_\phi(V) = \left(\beta - \frac{1}{3}\right) f(V) + \rho_0 V^{-1}, \tag{38}$$

$$q(V) = \frac{9}{2}\left[\beta + \frac{\rho_0}{V f(V)}\right] - 1, \tag{39}$$

with

$$f(V) = cV^{-3\beta} + \frac{3\rho_0 V^{-1}}{1 - 3\beta}. \tag{40}$$

Imposing the condition of the accelerated expansion of the universe, ($q < 0$), on Eq. (34) and using (30), for $\alpha < \frac{3}{4}$, the following constraint on the parameter β is obtained

$$0 < \beta < \frac{\left(2 - \dfrac{16\alpha}{3}\right)\left(1 + \dfrac{\rho_\phi + \rho_{\rm GB}}{\rho_m}\right) - 3}{9\left(1 + \dfrac{\rho_\phi + \rho_{\rm GB}}{\rho_m}\right)}. \tag{41}$$

Assuming $\frac{\rho_\phi+\rho_{\rm GB}}{\rho_m} \approx \frac{7}{3}$, it is found

$$0 < \beta < 0.12 - 0.59\alpha. \tag{42}$$

For $\alpha = 0$, the above constraint condition reduces to $0 < \beta < 0.12$ which is just the same as what has been obtained in Ref. 13 in the absence of the GB term. For $c = 0$, from Eq. (28) the potential energy takes the following exponential form

$$U(\phi) = \frac{-32\alpha + 27\beta + 4\alpha(9\beta + 16\alpha)}{18 - 40\alpha - 54\beta}\rho_0 e^{-\frac{\phi}{\sqrt{\beta}}}, \tag{43}$$

where in the limit $\phi \to \infty$, $U(\phi) \to 0$. The GB coupling parameter is given by

$$\xi(\phi) = \frac{\alpha}{27\beta\rho_0}(20\alpha + 27\beta - 9)e^{\frac{\phi}{\sqrt{\beta}}}, \tag{44}$$

which has an exponential form too.

4. Results

As is seen, the solutions of the gravitational field equations depend on three arbitrary numerical parameters including an integration constant and the coefficients α and β. α and β range of values are fixed by considering the accelerated expansion of the universe state.

4.1. *Scalar-GB gravity*

The time variation of the deceleration parameter as a function of time for $\alpha = 0.1$ and different values of β has been plotted in Fig. 1. Figure 2 shows the situation where $\beta = 0.05$ and α has different values. The figure shows a transition from

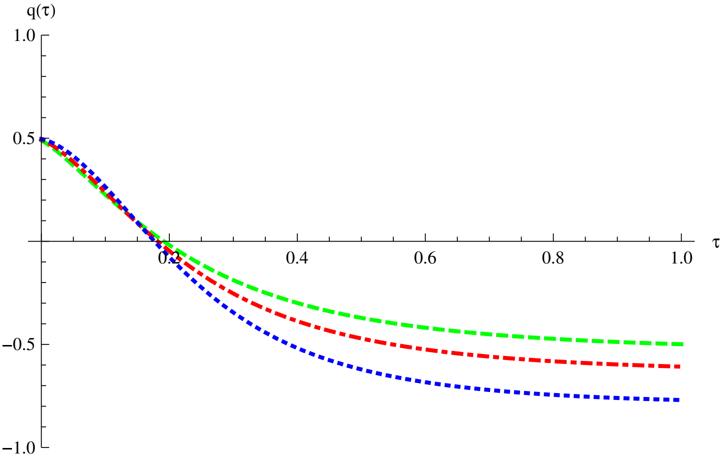

Fig. 1. Variation of the deceleration parameter q as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.02$ (dotted curve), $\beta = 0.06$ (dashed curve) and $\beta = 0.04$ (dot-dashed curve) with $\alpha = 0.1$ and $\frac{c}{\rho_0} = 30$.

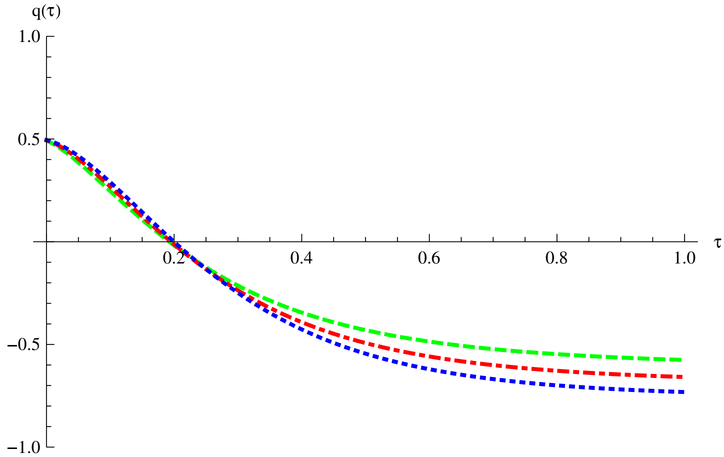

Fig. 2. Variation of the deceleration parameter q as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\alpha = 0.01$ (dotted curve), $\alpha = 0.09$ (dashed curve) and $\alpha = 0.05$ (dot-dashed curve) with $\beta = 0.05$ and $\frac{c}{\rho_0} = 30$.

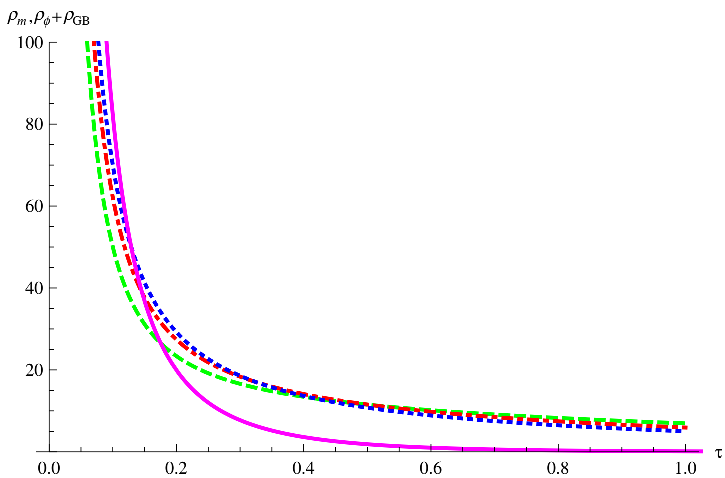

Fig. 3. Variation of the matter energy density ρ_m (solid curve) and the dark energy density $\rho_\phi + \rho_{\rm GB}$ as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.06$ (dotted curve), $\beta = 0.02$ (dashed curve) and $\beta = 0.04$ (dot-dashed curve) with $\alpha = 0.1$ and $\frac{c}{\rho_0} = 30$.

a deceleration epoch to an acceleration epoch in agreement with observations. In Fig. 3, the energy density of matter is compared with the energy density of the mixture of quintessence field and GB term for $\alpha = 0.1$ and different values of β. The figure shows that for early-time the matter energy density dominates the dark energy density $\rho_m > \rho_\phi + \rho_{\rm GB}$ but for late-time the dark energy density is dominated $\rho_\phi + \rho_{\rm GB} > \rho_m$ and has the main role in determining the dynamics of the universe. Figure 4 shows the comparison between the matter energy density

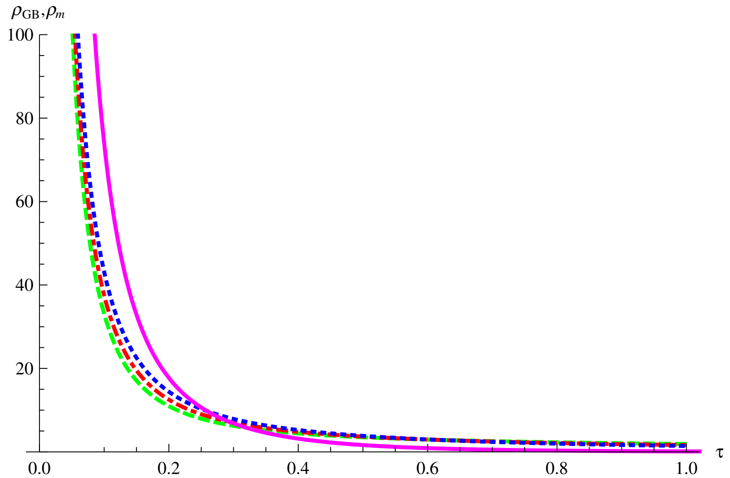

Fig. 4. Variation of the matter energy density ρ_m (solid curve) and the GB energy density $\rho_{\rm GB}$ as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.06$ (dotted curve), $\beta = 0.02$ (dashed curve) and $\beta = 0.04$ (dot-dashed curve) with $\alpha = 0.1$ and $\frac{c}{\rho_0} = 30$.

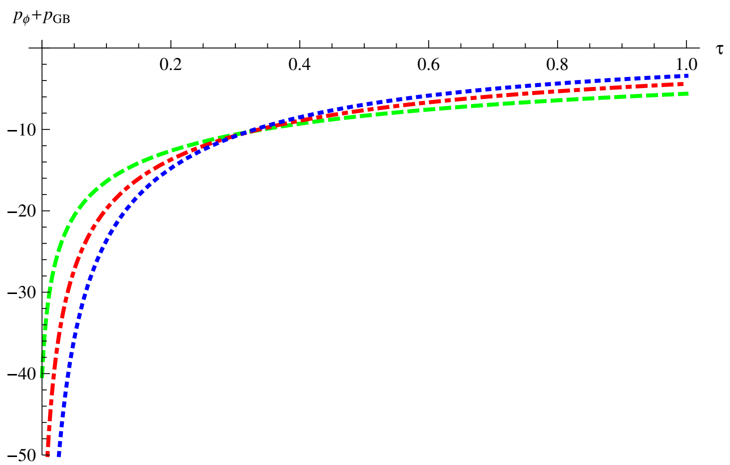

Fig. 5. Variation of the dark energy pressure $p_\phi + p_{\rm GB}$ as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.06$ (dotted curve), $\beta = 0.02$ (dashed curve) and $\beta = 0.04$ (dot-dashed curve) with $\alpha = 0.1$ and $\frac{c}{\rho_0} = 30$.

and the GB energy density. As is seen, the GB energy density is dominated and the dynamics of the universe is determined by the GB term. The evolution of the dark energy pressure, $p_{\rm de} = p_\phi + p_{\rm GB}$, has been plotted in Fig. 5. It shows that for all times the dark energy pressure is negative, which is the cause of the accelerated expansion of the universe. Figure 6 shows the time variation of the GB coupling parameter.

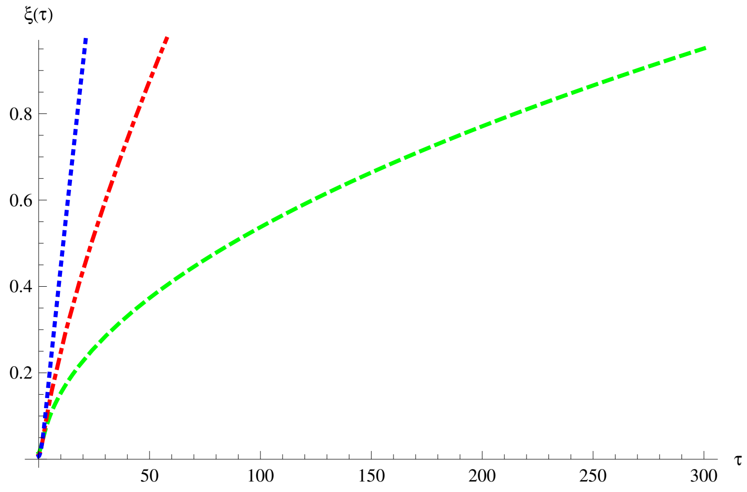

Fig. 6. Variation of the GB coupling parameter ξ as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.06$ (dotted curve), $\beta = 0.02$ (dashed curve) and $\beta = 0.04$ (dot-dashed curve) with $\alpha = 0.1$ and $\frac{c}{\rho_0} = 30$.

4.2. *Comparison of the scalar-GB with the quintessence field*

Now let us study the behavior of the deceleration parameter q, the scalar field energy density ρ_ϕ, and the pressure p_ϕ when the GB term vanishes and the model reduces to the scalar field model.[13] Figure 7 shows the behavior of the dust matter energy density and the quintessence field for different values of β. Although the

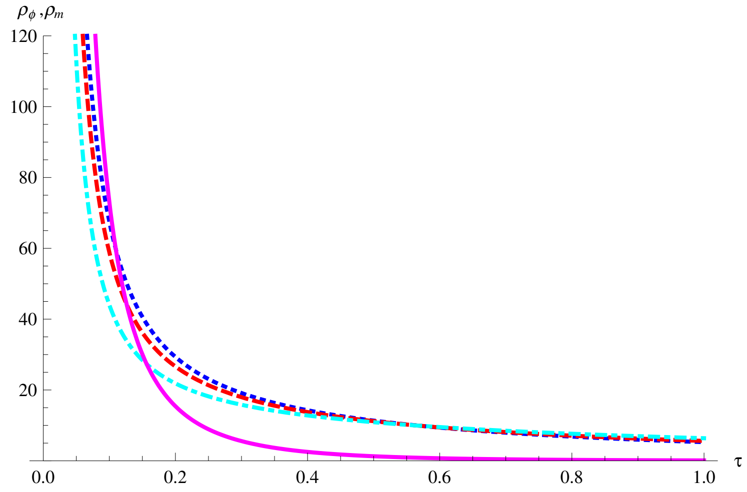

Fig. 7. Variation of the matter energy density ρ_m (solid curve) and the scalar field energy density ρ_ϕ as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.11$ (dotted curve), $\beta = 0.09$ (dashed curve) and $\beta = 0.07$ (dot-dashed curve) without GB term ($\alpha = 0$) and $\frac{c}{\rho_0} = 30$.

matter energy density is insensitive to β, the energy density of the quintessence field is sensitive to this free parameter. For early-time, the matter energy density is dominated; but for late-time, the quintessence energy density is dominated and determines the dynamics of the universe. The behavior of the deceleration parameter for different values of β has been shown in Fig. 8; a transition from decelerated phase to the accelerated phase is seen at late times. Figure 9 shows that

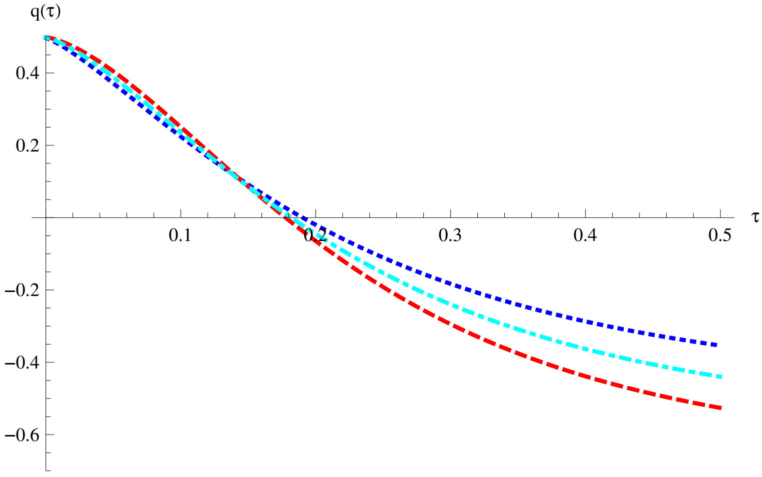

Fig. 8. Variation of the deceleration parameter q as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.11$ (dotted curve), $\beta = 0.07$ (dashed curve) and $\beta = 0.09$ (dot-dashed curve) without GB term ($\alpha = 0$) and $\frac{c}{\rho_0} = 30$.

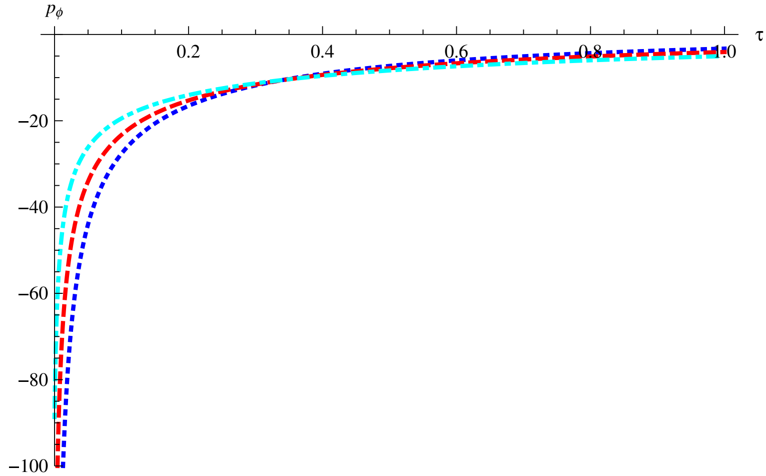

Fig. 9. Variation of the pressure of the scalar field p_ϕ as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.11$ (dotted curve), $\beta = 0.09$ (dashed curve) and $\beta = 0.07$ (dot-dashed curve) without GB term ($\alpha = 0$) and $\frac{c}{\rho_0} = 30$.

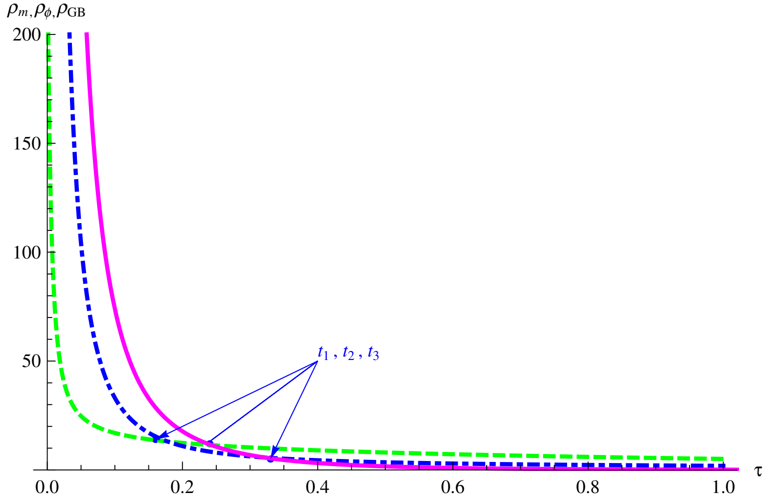

Fig. 10. Variation of the matter energy density ρ_m (solid curve), the scalar field energy density ρ_ϕ (dashed curve) and of the GB invariant $\rho_{\rm GB}$ (dot-dashed curve) as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.02$ with $\alpha = 0.1$ and $\frac{c}{\rho_0} = 30$.

the pressure is negative for all times; this justifies the accelerated expansion of the universe.[13]

Figure 10 shows the comparison between the matter energy density, the scalar field energy density and the GB energy density for a chosen value of the integration constant. During the first time interval $t < t_1$, the matter energy density is

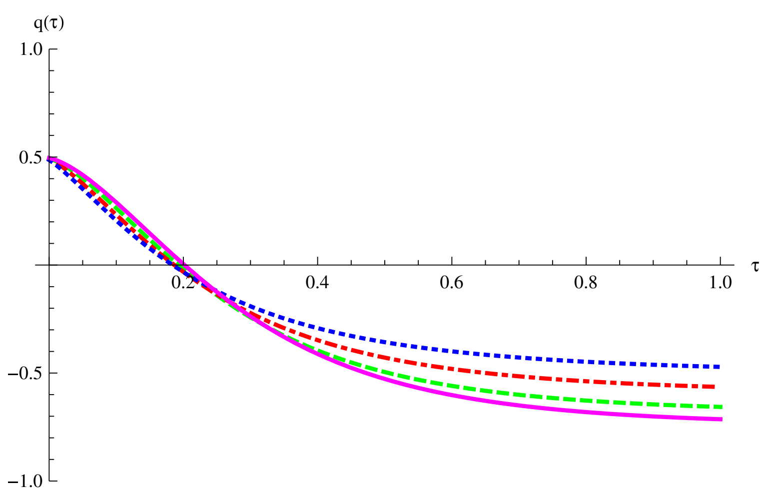

Fig. 11. Variation of the deceleration parameter q as a function of the time $\tau = \sqrt{\rho_0}(t - t_0)$ for $\beta = 0.11$ (dotted curve), $\beta = 0.07$ (dashed curve) and $\beta = 0.09$ (dot-dashed curve) without GB term ($\alpha = 0$) and for $\alpha = 0.1$ and $\beta = 0.02$ (solid line) in the scalar-GB gravity with $\frac{c}{\rho_0} = 30$.

dominated and the role of $\rho_{\rm GB}$ is more important than ρ_ϕ, $\rho_\phi < \rho_{\rm GB} < \rho_m$, whereas in the last time $t > t_3$, the energy density of the scalar field is dominated and the role of ρ_ϕ is more important than $\rho_{\rm GB}$, $\rho_m < \rho_{\rm GB} < \rho_\phi$. The behavior of the deceleration parameter in the scalar-GB gravity and the quintessence field has been shown in Fig. 11 in which a clear transition from decelerated to accelerated phase of the universe is seen.

5. Conclusions

In the present paper, we have studied a 4D action including the Einstein–Hilbert action and the GB curvature invariant coupled to the scalar field. Considering the gravitational field equation in the parametric form of the volume scale factor, V, we have obtained differential equation (17) in terms of the free functions ϕ and ξ. Specifying ϕ and ξ in term of V, we have found the physical quantities of the model in exact parametric forms. It has been shown that the current observationally confirmed accelerated phase of the universe and its transition from deceleration phase at early times can be explained by choosing appropriate and acceptable values of the constants and the parameters in the model under consideration. Finally, we should mention that the model presented here differs from the model introduced in Refs. 45 and 46. In these references, a power-law dependence of the scale factor $a(t) = a_0 t^\alpha$ has been considered and thus the deceleration parameter is constant, whereas in the present paper we have obtained the evolution of the declaration parameter as a function of time, $q(t)$. Moreover, here we have only considered the GB gravity coupled to the scalar field; the study of the modified GB gravity theories, $f(G)$, with and without the scalar field can be considered as the subject of future investigations.

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